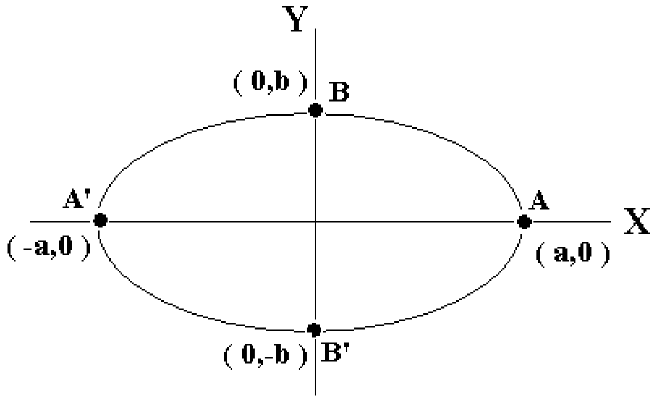
B) Ellipse

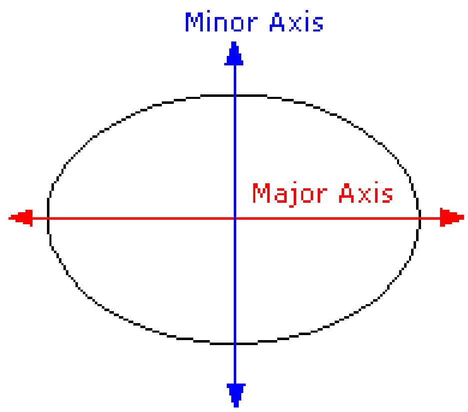
The general equations of an ellipse is very similar to that of the circle: we could write the equation of the circle as $\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$, and because the ellipse is longer in one direction than the other, the divided r^2 value beneath the x^2 is different than the one under the y^2.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

a > b > 0
Centre = (0, 0)
Major axis = 2a (distance btwn horizontal edges)
Minor axis = 2b



** If b > a > 0, then the major axis is 2b, minor axis is 2a.*



Translation: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Centre = (h, k)

<u>**COMPLETING THE SQUARE FOR AN ELLIPSE**</u>
Examples:

1) Complete the square for the equation $x^2 - 4x + 4y^2 = 0$ and graph the ellipse. Label the main features (centre, horizontal edges, vertical edges).

$$x^2 - 4x + 4y^2 = 0$$
$$\left[x^2 - 4x + (\frac{-4}{2})^2\right] + 4y^2 = 0 + (\frac{-4}{2})^2$$
$$[x^2 - 4x + 4] + 4y^2 = 0 + 4$$
$$(x-2)^2 + 4y^2 = 4$$
$$\frac{(x-2)^2}{4} + \frac{4y^2}{4} = 1$$
$$\frac{(x-2)^2}{4} + \frac{y^2}{1} = 1$$
$$\frac{(x-2)^2}{2^2} + \frac{y^2}{1^2} = 1$$

So the centre is (2, 0), a = 2, b = 1.
Horizontal edges = (4, 0) and (0, 0).
Vertical edges = (2, 1) and (2, -1).

2) Complete the square for the equation $x^2 + 4y^2 - 6x + 16y + 21 = 0$ and graph the ellipse. Label the main features (centre, horizontal edges, vertical edges).

$$x^2 + 4y^2 - 6x + 16y + 21 = 0$$
$$[x^2 - 6x] + [4y^2 + 16y] = -21$$
$$[x^2 - 6x] + 4[y^2 + 4y] = -21$$
$$\left[x^2 - 6x + (\frac{-6}{2})^2\right] + 4\left[y^2 + 4y + (\frac{4}{2})^2\right] = -21 + (\frac{-6}{2})^2 + 4(\frac{4}{2})^2$$
$$[x^2 - 6x + 9] + 4[y^2 + 4y + 4] = -21 + 9 + 4 \cdot 4$$
$$(x-3)^2 + 4(y+2)^2 = 4$$
$$\frac{(x-3)^2}{4} + \frac{4(y+2)^2}{4} = \frac{4}{4}$$
$$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{1} = 1$$
$$\frac{(x-3)^2}{2^2} + \frac{(y+2)^2}{1^2} = 1$$

So the centre is (3, -2), a = 2, b = 1.
Horizontal edges = (5, -2) and (1, -2).
Vertical edges = (3, -1) and (3, -3).

3) Complete the square for the equation $3x^2 - 18x + 5y^2 - 20y + 32 = 0$ and graph the ellipse. Label the main features (centre, horizontal edges, vertical edges).

$$3x^2 - 18x + 5y^2 - 20y + 32 = 0$$
$$3[x^2 - 6x] + 5[y^2 - 4y] = -32$$
$$3\left[x^2 - 6x + (\frac{-6}{2})^2\right] + 5\left[y^2 - 4y + (\frac{-4}{2})^2\right] = -32 + 3(\frac{-6}{2})^2 + 5(\frac{-4}{2})^2$$
$$3[x^2 - 6x + 9] + 5[y^2 - 4y + 4] = -32 + 3 \cdot 9 + 5 \cdot 4$$
$$3(x-3)^2 + 5(y-2)^2 = 15$$
$$\frac{3(x-3)^2}{15} + \frac{5(y-2)^2}{15} = \frac{15}{15}$$
$$\frac{(x-3)^2}{5} + \frac{(y-2)^2}{3} = 1$$
$$\frac{(x-3)^2}{(\sqrt{5})^2} + \frac{(y-2)^2}{(\sqrt{3})^2} = 1$$

So the centre is (3, 2), a = $\sqrt{5}$, b = $\sqrt{3}$.
Horizontal edges = (3 + $\sqrt{5}$, 2) and (3 - $\sqrt{5}$, 2).
Vertical edges = (3, 2 + $\sqrt{3}$) and (3, 2 - $\sqrt{3}$).

Worksheet
Delta Ex 37.3 pg 362 Q1, 2, 3, 5, 8 (ignore foci)
Extension: Ex 37.3 Q4, 6, 7, 9